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岩土流动势的存在性及解耦性

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摘 要: 基于不可逆热力学岩土损伤理论的流动势理论存在两个基本问题: (1) 岩土材料是否存在流动势; (2) 流动势可否分解为塑性势和损伤势, 即塑性、损伤两种机制可否解耦, 给出了流动势存在的充分必要条件并加以严格证明, 结合岩土的基本变形性质, 证明了岩土不存在流动势。进一步考察流动势的解耦性, 给出了流动势的塑性流动与损伤流动可解耦的充分必要条件, 并进行了严格论证, 结合土体的基本变形性质, 证明了岩土塑性流动和损伤流动机制不可解耦, 从而也为其他固体材料流动势的存在性和解耦性提供判别依据。

关 键 词: 岩土材料; 本构关系; 不可逆热力学; 损伤; 流动势

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Existence and decoupling for flow potential of geomaterials

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Abstract: Based on irreversible thermodynamics, the paper elaborates two fundamental constitutive issues: (1) existence or inexistence of the flow potential; (2) the flow potential can or cannot be decomposed into a damage potential and a plastic potential, namely, whether the two mechanisms can be decoupled. Within this framework, sufficient and necessary condition for the existence and the decoupling of the flow potential are given, and they are strictly proved in theory. According to the mechanical property of the geomaterial, the inexistence and the un-decoupling of the flow potential are also proved; meanwhile, it is shown that the existence and decoupling of other solid materials can theoretically be obtained from the two fundamental theories.

Key words: geomaterial; constitutive relation; irreversible thermodynamics; damage; flow potential

1 引 言

基于不可逆热力学的损伤力学是固体力学的一个重要分支, 它利用不可逆热力学和连续介质力学的唯象学方法, 研究材料损伤的力学过程^[1-5]。但在其基本理论方面仍然存在尚未解决的本质性问题。目前, 基于不可逆热力学的损伤理论普遍通过使用流动势 (flow potential) 来确定塑性和损伤的内变量发展方向, 而流动势的概念一开始是类比塑性力学中的塑性势而得到^[6-12]。因此, 流动势的存在性条件还不清楚, 也不清楚塑性流动与损伤流动解耦的适用条件, 致使在流动势的使用上产生一定程度的混乱: 部分学者^[13-16]采用塑性与损伤解耦的方法, 将损伤和塑性分解成两个独立的流动势面,

分别描述塑性和损伤的发展; 还有学者^[17-22]则采用塑性与损伤耦合的方法, 用一个光滑的广义流动势面来描述塑性和损伤的演化。

近年来, 基于不可逆热力学的损伤理论用于岩土材料领域^[23-31], 岩土材料属于多孔、多相散粒状材料并呈现出力学机制的复杂性和特殊性, 必然与固体连续材料之间具有很大的差异。因此, 对损伤理论的基本问题的论证变得更为迫切和必要。本文拟从连续介质力学出发, 严格论证流动势的存在性条件和塑性流动与损伤流动可解耦的条件, 并结合岩土材料的基本力学变形机制, 给出岩土流动势是否存在及是否可解耦的严格证明。对岩土损伤理论的基本问题进行深入的研究, 对建立具有科学理论依据的岩土损伤本构模型具有十分重要的意义。

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2 热力学基础

2.1 热力学第一定律

自然界中一切物质都具有能量。能量虽然有各种不同的形式,并且可以从一种形式转化为另一种形式,从一个物体传递给另一个物体,但在转化和传递过程中,能量的数量是守恒的。这一定律的重要意义在于:当热力学体系处于平衡态时,有一个称之为内能的状态函数^[32]。在绝热过程中,体系内能的增加等于外界对该体系所做的功。具体地,设物体的内能 U 和动能 K 为

$$\left. \begin{aligned} U &= \int_V \rho u dv \\ K &= \frac{1}{2} \int_V \rho \mathbf{v} \cdot \mathbf{v} dv \end{aligned} \right\} \quad (1)$$

式中: ρ 为物质密度; u 为内能密度; $\mathbf{v}$ 为速度向量; V 为物体的空间域。

类似地,外力对物体做的增量功 dW 为

$$dW = \int_V \mathbf{f} dv + \oint_{\partial V} \mathbf{p} n dA \quad (2)$$

环境传给物体的热量的增量 dQ 为

$$dQ = \int_V \rho r dv - \oint_{\partial V} \mathbf{q} n dA \quad (3)$$

式中: $\mathbf{f}$ 为体力向量; $\mathbf{p}$ 为面力向量; r 为热源强度; $\mathbf{q}$ 为热通量向量; ∂V 为物体表面; dA 为面元; $\mathbf{n}$ 为 ∂V 的外法向单位向量。

由热力学第一定律,有

$$dK + dU = dW + dQ \quad (4)$$

根据能量守恒定律的局部形式,由 V 的任意性,有

$$\rho du - \rho r + \operatorname{div} \mathbf{q} - \sigma : d\boldsymbol{\varepsilon} = 0 \quad (5)$$

2.2 热力学第二定律

热力学第二定律主要对过程自发进行的方向给予说明,能量转换和传递的热力学过程可分为“正过程”和“逆过程”两类,前者可以“自发”进行,而后者必须伴随“正过程”才能实现。以上关于热力学过程进行方向的定律有许多相互等价的表述,其 Clausius 说法为:不可能把热量从低温物体传给高温物体而不产生其他影响。

热力学第二定律的 Clausius-Duhem 不等式形式为

$$dS - Q_T \geq 0 \quad (6)$$

其中: $S = \int_V \rho s dv$, $Q_T = \int_V \frac{\rho r}{T} dv - \oint_{\partial V} \frac{\mathbf{q}}{T} n dA$ 。

式中: S 为熵; T 为绝对温度。

又根据散度定理,有:

$$\oint_{\partial V} \frac{\mathbf{q}}{T} n dA = \int_V \operatorname{div} \left(\frac{\mathbf{q}}{T} \right) dv = \int_V \left(\frac{1}{T} \operatorname{div} \mathbf{q} - \frac{\mathbf{q}}{T^2} \cdot \operatorname{grad} T \right) dv \geq 0 \quad (7)$$

式中: grad 为梯度算子。

将式 (5) 带入式 (7), 有局部形式为

$$\sigma : d\boldsymbol{\varepsilon} - \rho (du - T dS) - \frac{\mathbf{q}}{T} \cdot \operatorname{grad} T \geq 0 \quad (8)$$

在整个分析过程中, ρ 作为乘子出现,如果所有的广延量从每单位质量转化为每单位体积,则 ρ 在公式中消失,从而简化了符号标注^[33]。下文采用这一记法。

2.3 热力学势及耗散不等式

热力学势是一组独立的状态变量的函数。此类函数只与独立状态变量的值有关,而与变化的过程(历史)无关^[34]。独立状态变量包括塑性应变 $\boldsymbol{\varepsilon}^p$ 和损伤 D , 与独立状态相关联的是相应的热力学力。一般地,热力学势可表示为 $\Pi = \Pi(\chi_0, \chi_1, \dots)$, 从而

$$\text{可得 } d\Pi = \frac{\partial \Pi}{\partial \chi_0} d\chi_0 + \frac{\partial \Pi}{\partial \chi_1} d\chi_1 + \dots$$

上式是热力学势的一个普遍表达式,其中 $\frac{\partial \Pi}{\partial \chi_i}$

是 χ_i 对应的热力学力。

具体地,以 ψ 表示自由能,则有 $\psi = u - TS$, 那么

$$d\psi = du - T dS - dTS \quad (9)$$

将式 (9) 代入式 (5)、式 (8) 中,有

$$\sigma : d\boldsymbol{\varepsilon} - (d\psi + S dT) - T dS + r - \operatorname{div} \mathbf{q} = 0 \quad (10)$$

$$\sigma : d\boldsymbol{\varepsilon} - (d\psi + S dT) - \frac{1}{T} \mathbf{q} \cdot \operatorname{grad} T \geq 0 \quad (11)$$

由上面的热力学势的讨论可知,热力学系统 ψ 应是外变量(应变 $\boldsymbol{\varepsilon}^e$ 和绝对温度 T) 和内变量(塑性内变量 k 和损伤变量 D) 的函数。因此, ψ 可写为 $\psi = \psi(\boldsymbol{\varepsilon}^e, k, D, T)$,

并且

$$d\psi = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e} : d\boldsymbol{\varepsilon}^e + \frac{\partial \psi}{\partial k} : dk + \frac{\partial \psi}{\partial D} : dD + \frac{\partial \psi}{\partial T} : dT$$

因此,有

$$\left(\sigma - \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e} \right) : d\boldsymbol{\varepsilon}^e + \sigma : d\boldsymbol{\varepsilon}^p - \frac{\partial \psi}{\partial k} dk - \frac{\partial \psi}{\partial D} dD - \left(S + \frac{\partial \psi}{\partial T} \right) dT - T dS + r - \operatorname{div} \mathbf{q} = 0 \quad (12)$$

$$\left(\sigma - \frac{\partial \psi}{\partial \varepsilon^e} \right) : d\varepsilon^e + \sigma : d\varepsilon^p - \frac{\partial \psi}{\partial k} dk - \frac{\partial \psi}{\partial D} dD - \left(S + \frac{\partial \psi}{\partial T} \right) dT + qg \geq 0 \quad (13)$$

式中: $g = \frac{1}{T} \text{grad} T$ 。

由 $d\varepsilon^e$ 和 dT 的任意性, 那么根据上式, 取

$$\sigma = \frac{\partial \psi}{\partial \varepsilon^e}, S = -\frac{\partial \psi}{\partial T},$$

定义热力学广义力 K 、 Y 分别为

$$K = -\frac{\partial \psi}{\partial k}, Y = -\frac{\partial \psi}{\partial D}$$

则式 (12)、(13) 可表示为

$$\sigma : d\varepsilon^p + Kdk + YdD - TdS + r - \text{div} q = 0 \quad (14)$$

$$\sigma : d\varepsilon^p + Kdk + YdD + qg \geq 0 \quad (15)$$

岩土材料一般忽略热耗散, 由式 (14)、(15) 得 Clausius-Duhem 不等式为

$$\sigma : d\varepsilon^p + Kdk + YdD \geq 0 \quad (16)$$

3 流动势函数

式 (16) 即为热力学中的 Clausius-Duhem 耗散不等式, 因此, 对于任意材料具有普适性。大量文献将式 (16) 作为耗散势的定义, 然后通过 Legendre-Fenchel 变换, 将耗散势变换为与其对偶的流动势。显而易见, 若耗散势不存在, 则 Legendre-Fenchel 变换失效, 无法得到流动势。文献[35]已分析式 (16) 仅为耗散能, 不一定是耗散势, 其为耗散势是有条件的, 并给出了耗散势的一般表达式及存在性条件, 而岩土不满足耗散势存在的条件, 即耗散势不存在, 也就意味着无法通过 Legendre-Fenchel 变换得到流动势。部分学者越过耗散势, 直接类比塑性力学中的塑性势而得到流动势的概念, 也看不出有严格的理论依据。因此, 流动势的存在性是需要论证的。

3.1 流动势存在性条件及其证明

内变量一般的演化方程为

$$\left. \begin{aligned} d\varepsilon_v^p &= d\lambda m(\sigma, K, Y; \varepsilon^p, k, D) \\ d\varepsilon_s^p &= d\lambda n(\sigma, K, Y; \varepsilon^p, k, D) \\ d\varepsilon_{\theta_\sigma}^p &= d\lambda r(\sigma, K, Y; \varepsilon^p, k, D) \\ dk &= d\lambda h(\sigma, K, Y; \varepsilon^p, k, D) \\ dD &= d\lambda t(\sigma, K, Y; \varepsilon^p, k, D) \end{aligned} \right\} \quad (17)$$

若存在流动势 Q , 使

$$\left. \begin{aligned} m(\sigma, K, Y; \varepsilon^p, k, D) &= \frac{\partial Q}{\partial p} \\ n(\sigma, K, Y; \varepsilon^p, k, D) &= \frac{\partial Q}{\partial q} \\ r(\sigma, K, Y; \varepsilon^p, k, D) &= \frac{\partial Q}{\partial \theta_\sigma} \\ h(\sigma, K, Y; \varepsilon^p, k, D) &= \frac{\partial Q}{\partial K} \\ t(\sigma, K, Y; \varepsilon^p, k, D) &= \frac{\partial Q}{\partial Y} \end{aligned} \right\} \quad (18)$$

于是

$$\left. \begin{aligned} d\varepsilon_v^p &= d\lambda \frac{\partial Q}{\partial p} \\ d\varepsilon_s^p &= d\lambda \frac{\partial Q}{\partial q} \\ d\varepsilon_{\theta_\sigma}^p &= d\lambda \frac{\partial Q}{\partial \theta_\sigma} \\ dk &= d\lambda \frac{\partial Q}{\partial K} \\ dD &= d\lambda \frac{\partial Q}{\partial Y} \end{aligned} \right\} \quad (19)$$

其中 $d\lambda \geq 0$ 。

定理 1: 流动势函数 $Q = Q(p, q, \theta_\sigma, K, Y)$ 存在的充分必要条件是等式 $\frac{\partial n}{\partial p} = \frac{\partial m}{\partial q}$, $\frac{\partial r}{\partial p} = \frac{\partial n}{\partial \theta_\sigma}$,

$\frac{\partial h}{\partial p} = \frac{\partial m}{\partial K}$, $\frac{\partial t}{\partial p} = \frac{\partial m}{\partial Y}$, $\frac{\partial r}{\partial q} = \frac{\partial n}{\partial \theta_\sigma}$, $\frac{\partial h}{\partial q} = \frac{\partial n}{\partial K}$, $\frac{\partial t}{\partial q} = \frac{\partial n}{\partial Y}$, $\frac{\partial h}{\partial \theta_\sigma} = \frac{\partial r}{\partial K}$, $\frac{\partial t}{\partial \theta_\sigma} = \frac{\partial r}{\partial Y}$, $\frac{\partial h}{\partial K} = \frac{\partial t}{\partial Y}$, 在广义应力空间内恒成立。

必要性: 假设存在流动势 Q , 使得

$$m = \frac{\partial Q}{\partial p}, n = \frac{\partial Q}{\partial q}, r = \frac{\partial Q}{\partial \theta_\sigma}, h = \frac{\partial Q}{\partial K}, t = \frac{\partial Q}{\partial Y}$$

对于 $\frac{\partial Q}{\partial p} = m$, $\frac{\partial Q}{\partial q} = n$, 则有 $\frac{\partial^2 Q}{\partial q \partial p} = \frac{\partial m}{\partial q}$,

$$\frac{\partial^2 Q}{\partial p \partial q} = \frac{\partial n}{\partial p}$$

因为 $\frac{\partial^2 Q}{\partial q \partial p}$ 、 $\frac{\partial^2 Q}{\partial p \partial q}$ 连续, 所以

$$\frac{\partial^2 Q}{\partial q \partial p} = \frac{\partial^2 Q}{\partial p \partial q}, \text{ 即 } \frac{\partial n}{\partial p} = \frac{\partial m}{\partial q}$$

同理可得到:

$$\frac{\partial r}{\partial p} = \frac{\partial n}{\partial \theta_\sigma}, \frac{\partial h}{\partial p} = \frac{\partial m}{\partial K}, \frac{\partial t}{\partial p} = \frac{\partial m}{\partial Y}, \frac{\partial r}{\partial q} = \frac{\partial n}{\partial \theta_\sigma},$$

$$\frac{\partial h}{\partial q} = \frac{\partial n}{\partial K}, \quad \frac{\partial t}{\partial q} = \frac{\partial n}{\partial Y}, \quad \frac{\partial h}{q \partial \theta_\sigma} = \frac{\partial r}{\partial K}, \quad \frac{\partial t}{q \partial \theta_\sigma} = \frac{\partial r}{\partial Y},$$

$$\frac{\partial t}{\partial K} = \frac{\partial h}{\partial Y}.$$

充分性: 由 $\frac{\partial n}{\partial p} = \frac{\partial m}{\partial q}, \frac{\partial r}{\partial p} = \frac{\partial n}{q \partial \theta_\sigma}, \frac{\partial h}{\partial p} = \frac{\partial m}{\partial K},$
 $\frac{\partial t}{\partial p} = \frac{\partial m}{\partial Y}, \frac{\partial r}{\partial q} = \frac{\partial n}{q \partial \theta_\sigma}, \frac{\partial h}{\partial q} = \frac{\partial n}{\partial K}, \frac{\partial t}{\partial q} = \frac{\partial n}{\partial Y},$
 $\frac{\partial h}{q \partial \theta_\sigma} = \frac{\partial r}{\partial K}, \frac{\partial t}{q \partial \theta_\sigma} = \frac{\partial r}{\partial Y}, \frac{\partial t}{\partial K} = \frac{\partial h}{\partial Y},$ 在广义应力空间内恒成立。

可知, $\int_L mdp + ndq + rqd\theta_\sigma + hdK + tdY$ 在广义应变空间内与路径无关。从点 $(p_0, q_0, \theta_{\sigma 0}, K_0, Y_0)$ 到点 $(p, q, \theta_\sigma, K, Y)$ 的势耗散能可表示为

$$\int_{(p_0, q_0, \theta_{\sigma 0}, K_0, Y_0)}^{(p, q, \theta_\sigma, K, Y)} mdp + ndq + rqd\theta_\sigma + hdK + tdY \quad (20)$$

式 (20) 的值取决于终点 $M(p, q, \theta_\sigma, K, Y)$, 因此, 它是 $p, q, \theta_\sigma, K, Y$ 的函数, 把这函数记作 $Q(p, q, \theta_\sigma, K, Y)$, 即

$$Q = \int_{(p_0, q_0, \theta_{\sigma 0}, K_0, Y_0)}^{(p, q, \theta_\sigma, K, Y)} mdp + ndq + rqd\theta_\sigma + hdK + tdY \quad (21)$$

下面证明函数 $dQ = mdp + ndq + rqd\theta_\sigma + hdK + tdY$ 。因为 $p, q, \theta_\sigma, K, Y$ 都是连续的, 因此, 只要证明 $\frac{\partial Q}{\partial p} = m, \frac{\partial Q}{\partial q} = n, \frac{\partial Q}{q \partial \theta_\sigma} = r, \frac{\partial Q}{\partial K} = h, \frac{\partial Q}{\partial Y} = t$ 即可。

因为

$$Q = \int_{(p_0, q_0, \theta_{\sigma 0}, K_0, Y_0)}^{(p, q, \theta_\sigma, K, Y)} mdp + ndq + rqd\theta_\sigma + hdK + tdY =$$

$$\int_{p_0}^p mdp + \int_{q_0}^q ndq + \int_{\theta_{\sigma 0}}^{\theta_\sigma} rdK + \int_{K_0}^K hdK + \int_{Y_0}^Y tdY$$

所以

$$\frac{\partial Q}{\partial p} = \frac{\partial}{\partial p} \int_{p_0}^p mdp + \frac{\partial}{\partial p} \int_{q_0}^q ndq + \frac{\partial}{\partial p} \int_{\theta_{\sigma 0}}^{\theta_\sigma} rdK +$$

$$\frac{\partial}{\partial p} \int_{K_0}^K hdK + \frac{\partial}{\partial p} \int_{Y_0}^Y tdY = m$$

$$\text{类似地有 } \frac{\partial Q}{\partial q} = n, \frac{\partial Q}{q \partial \theta_\sigma} = r, \frac{\partial Q}{\partial K} = h, \frac{\partial Q}{\partial Y} = t$$

从而

$$dQ(p, q, \theta_\sigma, K, Y) = \frac{\partial Q}{\partial p} dp + \frac{\partial Q}{\partial q} dq + \frac{\partial Q}{q \partial \theta_\sigma} d\theta_\sigma +$$

$$\frac{\partial Q}{\partial K} dK + \frac{\partial Q}{\partial Y} dY$$

则:

$$Q = \int_{(p_0, q_0, \theta_{\sigma 0}, K_0, Y_0)}^{(p, q, \theta_\sigma, K, Y)} mdp + ndq + rqd\theta_\sigma + hdK + tdY$$

到此证明完毕。

3.2 岩土不满足流动势存在性条件的证明

由定理 1 的结论, 岩土材料是否存在流动势 Q , 只需考察等式: $\frac{\partial n}{\partial p} = \frac{\partial m}{\partial q}, \frac{\partial r}{\partial p} = \frac{\partial n}{q \partial \theta_\sigma}, \frac{\partial h}{\partial p} = \frac{\partial m}{\partial K},$
 $\frac{\partial t}{\partial p} = \frac{\partial m}{\partial Y}, \frac{\partial r}{\partial q} = \frac{\partial n}{q \partial \theta_\sigma}, \frac{\partial h}{\partial q} = \frac{\partial n}{\partial K}, \frac{\partial t}{\partial q} = \frac{\partial n}{\partial Y},$
 $\frac{\partial h}{q \partial \theta_\sigma} = \frac{\partial r}{\partial K}, \frac{\partial t}{q \partial \theta_\sigma} = \frac{\partial r}{\partial Y}, \frac{\partial t}{\partial K} = \frac{\partial h}{\partial Y},$ 在广义应力空间内是否恒成立。若所有等式在广义应力空间内恒成立, 则必存在流动势函数 Q 。反之, 若其中任一等式在广义应力空间内并不恒成立, 则必不存在流动势函数 Q 。

利用岩土材料的基本变形性质来验证其是否存在流动势, 假设存在流动势函数 Q , 则 $\frac{\partial Q(p, q, \theta_\sigma)}{\partial p} =$
 $m(p, q, \theta_\sigma), \frac{\partial Q(p, q, \theta_\sigma)}{\partial q} = n(p, q, \theta_\sigma), \frac{\partial Q(p, q, \theta_\sigma)}{q \partial \theta_\sigma} =$
 $r(p, q, \theta_\sigma)。$

对 $m(p, q, \theta_\sigma), n(p, q, \theta_\sigma), r(p, q, \theta_\sigma)$ 微分, 得:

$$\left. \begin{aligned} dm &= \frac{\partial m}{\partial p} dp + \frac{\partial m}{\partial q} dq + \frac{\partial m}{\partial \theta_\sigma} d\theta_\sigma = \\ &C_{11} dp + C_{12} dq + C_{13} d\theta_\sigma \\ dn &= \frac{\partial n}{\partial p} dp + \frac{\partial n}{\partial q} dq + \frac{\partial n}{\partial \theta_\sigma} d\theta_\sigma = \\ &C_{21} dp + C_{22} dq + C_{23} d\theta_\sigma \\ dr &= \frac{\partial r}{\partial p} dp + \frac{\partial r}{\partial q} dq + \frac{\partial r}{\partial \theta_\sigma} d\theta_\sigma = \\ &C_{31} dp + C_{32} dq + C_{33} d\theta_\sigma \end{aligned} \right\} \quad (22)$$

为了便于验证, 取特殊应力路径 (常规三轴试验)。对于常规三轴试验条件, 应力洛德角 θ_σ 始终为常数, 即 $d\theta_\sigma = 0$, 因此, 式 (22) 可简化为式 (23)、(24)。

$$\left. \begin{aligned} dm &= \frac{\partial m}{\partial p} dp + \frac{\partial m}{\partial q} dq = C_{11} dp + C_{12} dq \\ dn &= \frac{\partial n}{\partial p} dp + \frac{\partial n}{\partial q} dq = C_{21} dp + C_{22} dq \end{aligned} \right\} \quad (23)$$

或

$$\begin{Bmatrix} dm \\ dn \end{Bmatrix} = \begin{bmatrix} \frac{\partial m}{\partial p} & \frac{\partial m}{\partial q} \\ \frac{\partial n}{\partial p} & \frac{\partial n}{\partial q} \end{bmatrix} \begin{Bmatrix} dp \\ dq \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} dp \\ dq \end{Bmatrix} \quad (24)$$

由式(23)及图1所示, 对于应力增量方向①:
若 $dq=0$, $dp=1$, 则 $dm = \frac{\partial m}{\partial p} = C_{11}$, $dn = \frac{\partial n}{\partial p} = C_{21}$;
对于应力增量方向②: 若 $dp=0$, $dq=1$, 则
 $dm = \frac{\partial m}{\partial q} = C_{12}$, $dn = \frac{\partial n}{\partial q} = C_{22}$ 。

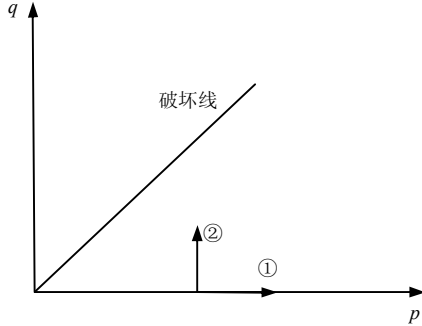


图1 应力增量方向
Fig.1 Directions of the stress increment

可见, 矩阵 $[C]$ 中任意元素的 C_{ij} 的物理意义为: 当微小单元体上仅作用有 j 方向的单位应力增量, 而其他方向无应力增量时, i 方向的塑性应变方向增量分量就等于 C_{ij} 。显然, 若存在流动势要求矩阵 $[C]$ 为对称阵。

对于金属材料, 始终对①有 $dn = \frac{\partial n}{\partial p} = C_{21} = 0$,

对②有 $dm = \frac{\partial m}{\partial q} = C_{12} = 0$ 。因此, 存在流动势 (即塑性势) $Q(p, q)$ 。

对于岩土材料, 如图1所示: 由于①使 $n \equiv 0$, 即 $dn = \frac{\partial n}{\partial p} = C_{21} = 0$, 而②有剪胀现象^[36-37], 必存在 $dm \neq 0$ 的状态, 即存在 $dm = \frac{\partial m}{\partial q} = C_{12} \neq 0$, 故 $\frac{\partial n}{\partial p} = \frac{\partial m}{\partial q}$ 不恒成立。因此, 不存在流动势 $Q(p, q)$, 进而不存在流动势 $Q(p, q, \theta_\sigma, K, Y)$ 。

4 流动势的解耦性

Lemaitre, Chaboche, Voyiadjis, Chow, Hansen, Zhu, Murakami, Nguyen, J.F. Shao 等^[1-2, 4, 13-16, 26-31]学者将 $\Omega(p, q, \theta_\sigma, K, Y)$ 分解, 分别构造了塑性流动势 $\Omega_p(p, q, \theta_\sigma, K)$ 与损伤流动势 $\Omega_d(Y)$ 。

定理 2: 流动势 $Q(p, q, \theta_\sigma, K, Y)$ 可分解为 $Q_p = Q_p(p, q, \theta_\sigma, K)$ 与 $Q_d = Q_d(Y)$ 的充分必要条件是等式 $\frac{\partial t}{\partial p} = 0$, $\frac{\partial m}{\partial Y} = 0$, $\frac{\partial t}{\partial q} = 0$, $\frac{\partial n}{\partial Y} = 0$, $\frac{\partial t}{\partial \theta_\sigma} = 0$,

$\frac{\partial r}{\partial Y} = 0$, $\frac{\partial t}{\partial K} = 0$, $\frac{\partial h}{\partial Y} = 0$ 在广义应力空间内恒成立。

必要性: 若 $Q(p, q, \theta_\sigma, K, Y) = Q_p(p, q, \theta_\sigma, K) + Q_d(Y)$, 则

$$\frac{\partial t}{\partial p} = \frac{\partial m}{\partial Y} = \frac{\partial^2 Q}{\partial Y \partial p} = \frac{\partial^2 (Q_p + Q_d)}{\partial Y \partial p} = \frac{\partial^2 Q_p}{\partial Y \partial p} + \frac{\partial^2 Q_d}{\partial Y \partial p} = 0$$

$$\frac{\partial t}{\partial q} = \frac{\partial n}{\partial Y} = \frac{\partial^2 Q}{\partial Y \partial q} = \frac{\partial^2 (Q_p + Q_d)}{\partial Y \partial q} = \frac{\partial^2 Q_p}{\partial Y \partial q} + \frac{\partial^2 Q_d}{\partial Y \partial q} = 0$$

$$\frac{\partial t}{\partial \theta_\sigma} = \frac{\partial r}{\partial Y} = \frac{\partial^2 Q}{\partial Y \partial \theta_\sigma} = \frac{\partial^2 (Q_p + Q_d)}{\partial Y \partial \theta_\sigma} = \frac{\partial^2 Q_p}{\partial Y \partial \theta_\sigma} + \frac{\partial^2 Q_d}{\partial Y \partial \theta_\sigma} = 0$$

$$\frac{\partial t}{\partial K} = \frac{\partial h}{\partial Y} = \frac{\partial^2 Q}{\partial Y \partial K} = \frac{\partial^2 (Q_p + Q_d)}{\partial Y \partial K} = \frac{\partial^2 Q_p}{\partial Y \partial K} + \frac{\partial^2 Q_d}{\partial Y \partial K} = 0$$

即等式 $\frac{\partial t}{\partial p} = 0$, $\frac{\partial m}{\partial Y} = 0$, $\frac{\partial t}{\partial q} = 0$, $\frac{\partial n}{\partial Y} = 0$, $\frac{\partial t}{\partial \theta_\sigma} = 0$, $\frac{\partial r}{\partial Y} = 0$, $\frac{\partial t}{\partial K} = 0$, $\frac{\partial h}{\partial Y} = 0$ 在广义应力空间内恒成立。

充分性:

结合 $Q(p, q, \theta_\sigma, K, Y)$ 存在的充分必要条件, 得

$$\frac{\partial n}{\partial p} = \frac{\partial m}{\partial q}, \quad \frac{\partial r}{\partial p} = \frac{\partial n}{\partial \theta_\sigma}, \quad \frac{\partial h}{\partial p} = \frac{\partial m}{\partial K}, \quad \frac{\partial r}{\partial q} = \frac{\partial n}{\partial \theta_\sigma},$$

$$\frac{\partial h}{\partial q} = \frac{\partial n}{\partial K}, \quad \frac{\partial h}{\partial \theta_\sigma} = \frac{\partial r}{\partial K}, \quad \frac{\partial t}{\partial p} = 0, \quad \frac{\partial m}{\partial Y} = 0, \quad \frac{\partial t}{\partial q} = 0,$$

$$\frac{\partial n}{\partial Y} = 0, \quad \frac{\partial t}{\partial K} = 0, \quad \frac{\partial h}{\partial Y} = 0 \text{ 在广义应力空间内恒成立。}$$

对于条件 $\frac{\partial t}{\partial p} = \frac{\partial m}{\partial Y} = 0$, 由 $m = \frac{\partial Q}{\partial p}$ 和 $t = \frac{\partial Q}{\partial Y}$, 可推得

$$\frac{\partial^2 Q}{\partial p \partial Y} = 0 \quad (25)$$

式(25)为一简单的二阶齐次偏微分方程, 对其求解只需直接积分, 得式(26)

$$Q = Q_1(p, q, \theta_\sigma, K) + Q_2(q, \theta_\sigma, K, Y) \quad (26)$$

对于条件 $\frac{\partial t}{\partial q} = \frac{\partial n}{\partial Y} = 0$ 可推得

$$\frac{\partial^2 Q}{\partial q \partial Y} = 0 \quad (27)$$

同理并结合式 (26), 得

$$Q = Q_1(p, q, \theta_\sigma, K) + Q_2(\theta_\sigma, K, Y) \quad (28)$$

对于条件 $\frac{\partial t}{q \partial \theta_\sigma} = \frac{\partial r}{\partial Y} = 0$ 可推得

$$\frac{\partial^2 Q}{q \partial \theta_\sigma \partial Y} = 0 \quad (29)$$

同理并结合式 (28), 得

$$Q = Q_1(p, q, \theta_\sigma, K) + Q_2(K, Y) \quad (30)$$

最后由条件 $\frac{\partial t}{\partial K} = \frac{\partial r}{\partial Y} = 0$ 可推得

$$\frac{\partial^2 Q}{\partial K \partial Y} = 0 \quad (31)$$

同理并结合式 (30), 最终得

$$Q = Q_1(p, q, \theta_\sigma, K) + Q_2(Y) \quad (32)$$

记 $Q_p(p, q, \theta_\sigma, K) = Q_1(p, q, \theta_\sigma, K)$, $Q_d(Y) = Q_2(Y)$,

$Q(p, q, \theta_\sigma, K, Y) = Q_p(p, q, \theta_\sigma, K) + Q_d(Y)$ 。

到此证明完毕。

由定理 2 知, 流动势的存在性是流动势解耦的前提条件, 而对于岩土材料不存在流动势。因此, 岩土材料塑性流动与损伤流动不可解耦。

5 结 语

本文基于不可逆热力学和连续介质力学, 以符合热力学第二定律的 Clausius-Duhem 耗散不等式为基础, 给出了流动势存在的充分必要条件并加以严格证明, 结合岩土的基本变形性质, 验证了岩土不存在流动势。并考察了流动势的解耦性, 给出了流动势塑性部分与损伤部分可解耦的充分必要条件并进行了严格论证, 得到了岩土材料不可解耦。对流动势基本问题的研究, 可以加深对流动势的认识, 为构建科学合理的损伤理论奠定基础。

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